A Predator-Prey Model for Dynamics of Cognitive Radios

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***Abstract*—Cognitive radio technology is well known to enhance spectrum utilization via opportunistic transmission at link level. However, the time dynamics of spectrum utilization among primary system users and secondary cognitive radio users in such cognitive radio networks remain unknown at this time. We note that the system behavior is very similar to interaction among different species coexisting in an ecosystem. Therefore, we take advantage of well-known predator-prey model, in which two species of predators species model, representing heterogeneous users, are fed by the preys, representing the communication resources. By analyzing the population dynamics in the spectrum-sharing ecosystem, we could analyze the transient behavior of the CRs and develop efficient ways to assess the stability of the CR system.**

***Index Terms*—Cognitive radio, ecology, predator-prey model, spectrum sharing, Lotka-Volterra model, biological ecosystem.**

I. Introduction

COGNITIVE radio (CR) has received a great amount of attentions to enhance spectrum utilization by spectrum sensing and opportunistic access. Unlicensed secondary users (SUs) adapt their transmissions/receptions to exploit the available spectrum resources while simultaneously limiting their interference with licensed primary users (PUs). In *interweave* paradigm [1], a zero-interference rationale is adopted, that is, SU can not interfere at all with the PU. This implies that SUs only use the spectrum that is not temporarily used by PUs and are obligated to evacuate the spectrum upon sensing primary transmission. In such heterogeneous environments with priorities to access spectrum, substantial efforts have been done to model the behaviors of PUs and SUs in a static fashion [2], [3]. Similar results are based on the analysis of a steady state system, that is, when system reaches its equilibrium. However, SUs are given the opportunities to utilize spectrum resources in a certain period of time, which the analysis through steady state based method may not be sufficient by ignoring transient system dynamics to reach equilibrium. More specifically, there may be difference between accumulated system performance and the result from steady state based methods. Thus, it is desirable to understand the time dynamic behaviors of PUs and SUs under possible access strategies to explore another direction to look into such problem. Biological approaches are regarded as an appropriate direction to analyze the time dynamics of user behaviors in heterogeneous environments [4]. For example, evolutionary game [5] is widely adopted to analyze the time dynamics of user behaviors in heterogeneous networks [6] or misbehaved SUs in cognitive radio network (CRN) [7]. However, the dynamics of the interactive behaviors between normal PUs and SUs have not been addressed yet and become our primary goal in this letter.

For interweave CR paradigm, the number (i.e., population) of the resources occupied by PUs is benefited from the loss in the population of the resources taken by SUs. As a result, this relationship between PUs and SUs is highly similar to the lion-coyote interaction in a natural environment. Moreover, the populations of both PUs and SUs can be benefited by the loss of the population of the radio resources, which implies that radio resources can be considered as preys to users. Therefore, traditional two-predator-one-prey model [8], a special of existing ecological models [9] [10], can be helpful in investigating the behaviors of PUs and SUs accessing radio resources through the trace of time. We consequently propose a population interaction approach to understand the dynamics of different species where the dynamics of PUs and SUs are modeled by a set of differential or difference equations. This proposed model could quickly identify time dynamics and behaviors of CRs under different parameter settings, which helps the development and deployment of CRNs.

II. Existing Ecological Model and Fitness Function

Classic Lotka-Volterra (L-V) model [9] describes relationship between population densities of one species of predator x_1 and one species prey x_2. Assuming that all the parameters in the model are positive, then

$$\begin{cases} \dot{x_1} = x_1(\alpha - \beta x_2) \\ \dot{x_2} = x_2(-\gamma + \delta x_1) \end{cases}, \tag{1}$$

where the parameters α and δ are respectively referred to the birth rate of the prey and the death rate of the predator, while β and γ reflect the predation between the predator and the prey. To understand the system from a more systematic point of view, [11] constructed the fitness landscape to reformulate the population model. For discrete difference equations, fitness H_i is defined as the change in population density X_i of a particular species i from t to $t+1$. The multiplicative nature of the unregulated growth puts population models into a special class of dynamics systems in which the right hand side of (2), representing the value of X_i in the next step, always includes a term for per capita growth rate, $1 + H_i$, multiplied by the current value of X_i. Thus we can give the equation $X_i(t+1) = [1+H_i(t)]X_i(t)$, while H_i may be a function of time or other factor we concerned due to the prior knowledge or further analysis to the system.

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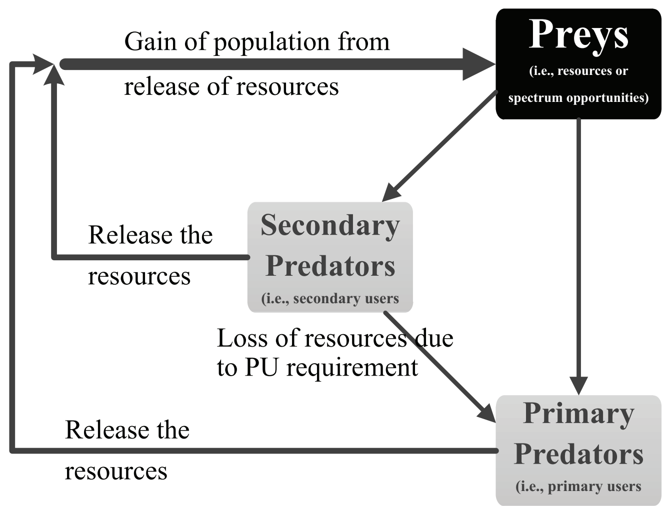

Fig. 1. The proposed model of two species of predators and one species of preys. The primary predators can consume the natural resource just as the primary users can use the radio resource with the highest priority. Primary predators can also "consume" the secondary predators. This phenomenon is similar to that in an interweave scenario, secondary users have to back off when the resource is requested by primary users.

III. Proposed Predator-Prey Model

Inspired by the biological systems in nature, we borrow the idea of describing eco-system with fitness functions to construct the model for CRs.

A. Description of the Model

As shown in Fig. 1, the ecological model describing the interweave paradigm of CRs consists of three types of population, that is, two species of predators and one species of preys. The primary predators, shown as the predators that can be fed on both secondary predators and preys, represent PUs, while the secondary predators are considered as SUs. The population (denote as $S(t)$) is defined as the number of active users at time t, where the term "active" is a state which a user can use the resources (i.e., spectrum opportunities) without interruption. When PUs try to access the resource that SUs are using, SUs have to give up the resources due to the nature of CRs. Such correspondence from predators to PUs and SUs is a reasonable inference from the nature of CR operation. PUs stand on the top of this "food chain" since the its population can be benefited from both taking away the resource owned by SUs or by occupying free radio resources (i.e., preys). However, unlike the common biological models, the total amount of resources in a communication system is often limited and fixed, which is important that this statement holds. The population model at time t is as follows: $S(t+1) = S(t) + G(t) - L(t)$, where $G(t)$ is a gain function related to the mechanism of population gain and $L(t)$ is the loss function related to traffic model of users. For instance, the active users in a medium access scenario are the users who have occupied a channel and successfully been transmitting, while the gain function $G(t)$ is related to the medium access protocol. When a PU tries to access a radio resource that is used by an SU, this SU immediately gives up the requested resource. This behavior forms a primary-secondary predator relation, while secondary predators are considered as another kind of resource to primary predators. The population of the preys corresponds to the number of empty channels. Consequently, the decrease in the population of preys (i.e., empty channels) implies that more users are currently using the channels.

B. Equilibrium of the Model

Without loss of generality, we assume that every user only uses one unit of time to transmit the packet. In other words, the packet size is small enough to be uploaded or downloaded within a unit of time. As a result, the loss function can be written as $L(t) = G(t-1)$. With some algebraic manipulations, the population iteration can be expressed as $S(t+1) = G(t)$. Obviously, $G(t)$ is related to the current population. Thus we may write $S(t+1) = G(S(t))$. The next issue of interest regarding system dynamics is the equilibrium of the model. It is straightforward that there exists at least one equilibrium S^* in such model if and only if the equation $S^* = G(S^*)$ has at least one solution. This is known as the fixed point problem in mathematic that has been well-studied [12]. Due to the fact that $S(t) \in [0, S_{max}]$ where S_{max} is the maximum number of users in the system, the set $[0, S_{max}]$ is a convex compact subset of an Euclidean space. It is reasonable that the maximum value of $G(t)$ does not exceed the maximum number of users, $G(S(t)) \in [0, S_{max}]$. As long as G is continuous in $[0, S_{max}]$, we can assure the existence of equilibrium.

IV. Dynamics of CRs

The proposed model is then applied to CR systems operating in interweave scenario [1]. The systems are consisting of PUs and SUs with N radio channels, where PUs and SUs have the random access probabilities p_1 and p_2, respectively. We suppose that each of PUs or SUs requires T time slots to transmit, which needs to start over again once being interrupted. In our mathematical evaluations, we set $T = 3$ to show that the transmission opportunity of an SU can be taken away in the middle of a transmission. The numbers of PUs and SUs occupying channels are denoted as S_1 and S_2, respectively. Denote the numbers of PUs and SUs trying to access the channels as C_P and C_S, respectively. Once a PU successfully occupies an empty channel (denoted as $S_C(t)$), the transmission completes without interruption. However, a SU is not able to finish a transmission if any PU accesses the channel currently utilized by the SU. For presenting of the dynamics of the system, the adopted measures for evaluations are channel occupance rates, which are highly related to the system throughput. The throughput of PUs, for instance, can be represented as $\frac{L^*(t)\left(\frac{M}{K}\right)}{\textbf{Channel Capacity}}$, where K is the length of a super frame, M is the actual packet size, and $L^*(t)$ is the value of loss function when the system reaches equilibrium.

Two access strategies are considered. With direct access, PUs and SUs take the exact same type of MAC protocol. Before actual accessing the channel, a user would perform spectrum sensing to see if there is anyone already occupying the channel. In this case, the user avoids the occupied one and finds another channel to access. In addition, SUs can only access the free channels, but PUs can access the channels even when SUs are already in use. When PUs and SUs are accessing channels at the same time, as interweave scenario suggests, SUs have to backoff if they choose a channel accessed by PUs. In other words, PUs and SUs are within the same collision domain for this scenario. If we alter the access strategy of SUs to sequential access, which SUs use direct radio resource allocation among themselves, SUs access the channel after PUs to avoid collision with PUs.

A. Direct Access

1) Population Gain by Predation: We now compute the gain in population by calculating the average numbers of empty channels occupied by PUs and SUs respectively. Assume that PUs take the channels uniformly, where the number of non-active PUs who are trying to access the empty channel is $\frac{S_C(t)}{S_C(t)+S_2(t)}$. The rest of the non-active PUs turn to the option of accessing the channels occupied by active SUs. The population gain of PUs from occupying available channels is

$$G_{PF}(t) = \frac{S_C(t)}{S_C(t)+S_2(t)} p_1 C_P(t) \left(1 - \frac{p_1}{S_C(t)}\right)^{C_P(t)-1}. \quad (2)$$

This suggests that SUs are "transparent" to PUs as one of the important features under the interweave CR dynamics. The gain of SUs from occupying an available channel is

$$G_S(t) = p_2 C_S(t) \left(1 - \frac{p_1}{S_C(t)}\right)^{\frac{S_2(t)}{S_C(t)+S_2(t)} p_1 C_P(t)} \cdot \left(1 - \frac{p_2}{S_C(t)}\right)^{C_S(t)-1}. \quad (3)$$

2) Population Gain from Predator-Predator Competition: The population of the active PUs can also benefit the predator-predator competition with SUs. By the assumption of taking channels uniformly, we can write the gain in the population of PUs by taking the channels that SUs have been using

$$G_{PS}(t) = \frac{S_2(t)}{S_C(t)+S_2(t)} p_1 C_P(t) \left(1 - \frac{p_1}{S_C(t)}\right)^{C_P(t)-1}. \quad (4)$$

3) Traffic Model: For the interweave CR scenario, once a PU successfully occupies an empty channel, the transmission completes without interruption, which implies that the gain of population in time t is exactly the loss of population in time $t+2$. On the other hand, the only way for an SU to finish a transmission is to be lucky enough that the transmission is not interrupted by PUs. The probability for an SU which successfully access the channel at time t to complete the second slot of transmission is $1-\frac{G_S(t)}{S_2(t+1)}$. Thus, the probability for an SU to complete a transmission can be represented as

$$\phi(t) = \left(1 - \frac{G_S(t)(1-\frac{G_S(t)}{S_2(t+1)})}{S_2(t+2)}\right). \quad (5)$$

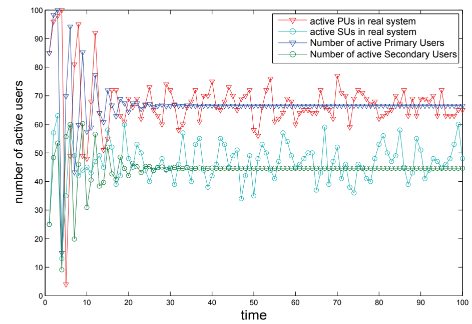

Fig. 2. The population dynamics of the system. The parameter setup is $T=3$, $N=200$, $p_1=0.8$, and $p_2=0.7$.

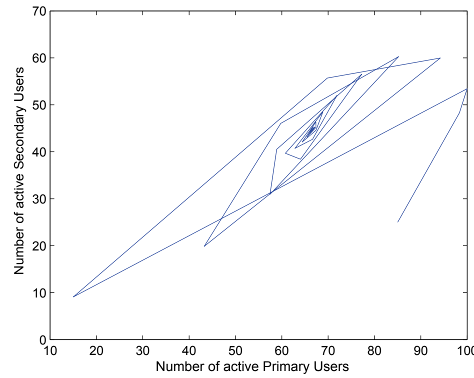

Fig. 3. The traditional 2-D phase diagram of the system. The parameter setup is same as that in Fig. 2. The equilibrium of population is given on the point of the center.

4) Population Dynamic Model: Now we can derive discrete fitness function in (2) of active SUs as

$$H_2(t) = \frac{[G_S(t) - G_{PS}(t) - \phi(t)G_S(t-2)]}{S_2(t)}. \quad (6)$$

Similar to above derivations, we also model the resources and PUs' resources fitness functions as

$$H_1(t) = \frac{[G_{PF}(t) + G_{PS}(t) - G_{PF}(t-2) - G_{PS}(t-2)]}{S_1(t)}. \quad (7)$$

The fitness function of the empty channels is

$$H_S(t) = [G_{PF}(t-2) + G_{PS}(t-2) - G_{PF}(t) - G_{PS}(t) - G_S(t) + \phi(t)G_S(t-2)]/S_C(t). \quad (8)$$

Fig. 2 illustrates the population dynamics with the comparison to a simulated system for the direct access scenario, which the proposed model is able to capture the time dynamic of the simulated system. The following parameters: $N=200$, $p_1=0.8$, and $p_2=0.7$ are used. The maximum numbers of the total PUs and SUs in the environment are 100 and 200, respectively. To prevent the system from a long convergence period, the parameters are designed to model a high-loaded environment where SUs have little available resources. From Fig. 2, we observe that the system converges to its equilibrium. The phase diagram describing the interaction between PUs

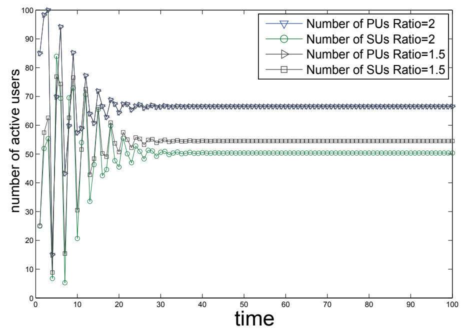

Fig. 4. The population dynamics in the scenario where only radio resource allocation among SUs is considered. The number of PUs in the simulation is 100 as the first part of the simulation. As we can see, the active rate of secondary users has a slightly enhancement by adapting our sequential access method in this case.

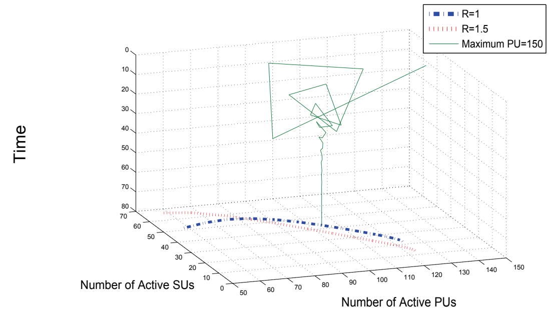

Fig. 5. The 3-D phase diagram in the scenario where only radio resource allocation among SUs is considered. The population will reach the equilibrium as the time evolving. The lines at the bottom indicates how the equilibrium shifts with the increase of users in the system.

and SUs is further illustrated in Fig. 3. As most predator-prey models, PUs and SUs are actually regulating to each other [5], which gives a simple illustration to the proposed model. Although the feature of the predator-prey system may fade because of the complicated three-layer structure, the results of Fig. 3 still follow the general behaviors of predator-prey in the nature, that is, the population of PUs would decrease when there exists too many active users in the environment. The system behavior along the time can be illustrated by the proposed model. By knowing the dynamics of a system, we can further explore the system limiting behaviors, and the impacts form different access protocols or a different strategies to enhance utility. For instance, SUs can conduct adaption of system parameters (i.e., equivalent to evolution) to improve the channel utilization.

B. Sequential access

In the following, we apply the model to consider a time slotted system using a different medium access strategy. The system is consisting of PUs and SUs with N channels, which have the access probability p_1 and p_2, respectively. However, the PUs and SUs do not access the spectrum resources at the same time in this system. The SUs try to access the spectrum resources after determination of radio resource to be occupied by PUs and avoid collisions with the PUs. In other words, we consider a direct adaption strategy of radio resource allocation among SUs to enhance the performance of the system. This can be easily achieved by adopting advanced features of CRs. The gain of SUs from occupying empty channels is changed as $G_S = p_2 C_S(t)\left(1 - \frac{p_2}{S_C(t) - G_{PF}(t)}\right)^{C_S(t)-1}$. As a result, the fitness of SUs becomes

$$H_2(t) = \frac{[G_S - G_{PS}(t) - \phi(t) G_S(t-2)]}{S_2(t)}. \quad (9)$$

Figs. 4 and 5 present the numerical illustrations of the system dynamics in this scenario, where the maximum number of PUs is the same as that in Figs. 2 and 3. We present the system dynamic in different PUs-to-SUs ratio R and alter the number of maximum users in the system. Two curves at the bottom in Fig. 5 shows the trend of the system equilibrium when we alter the number of maximum users in the system while using the same PU-to-SU ratio R. Even if we change the ratio R between PUs and SUs, the equilibrium of PUs does not change in our designed system, SUs are "transparent" to PUs, which is an important feature of CR. Another phenomenon taken into consideration is the comparison between sequential access and direct access. We observe that the avoidance of collision with PUs leads to an increase of the population equilibrium of the SUs, thereby enhancing the utility of the channels in the system. This is equivalent to avoiding interactions between SUs and PUs in the random access contention, thus the interaction only exists in the predation, which creates a more efficient spectrum sharing system.

This letter adopts ecological predator-prey model to capture the time dynamics of heterogenous users in CRs to present the process reaching the equilibrium in 3-D diagram to show the CR system dynamics. By adjusting the system parameters and using this analytical methodology, we wish to open the door of studying the untamed time dynamics of CRs, and to design the most effective CR or spectrum sharing system.

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